**G\*Power**

**http://www.psycho.uni-duesseldorf.de/abteilungen/aap/gpower3/user-guide-by-distribution/f/anova\_fixed\_effects\_special**

**ANOVA: Fixed effects, special, main effects and interactions**

This procedure may be used to calculate the power of main effects and interactions in ﬁxed effects ANOVAs with factorial designs. It can also be used to compute the power for planned comparisons. We will discuss both applications in turn.

**Main effects and interactions**

To illustrate the concepts underlying tests of main effects and interactions we will consider the speciﬁc example of an A × B × C factorial design, with *i* = 3 levels of *A*, *j* = 3 levels of *B*, and *k* = 4 levels of *C*. This design has a total number of 3 × 3 × 4 = 36 groups. A general assumption is that all groups have the same size and that in each group the dependent variable is normally distributed with identical variance.   
  
In a three factor design we may test three main effects of the factors *A*, *B*, *C*, three two-factor interactions *A* × *B*, *A* × *C* , *B* × *C*, and one three-factor interaction *A* × *B* × *C*. We write µijk for the mean of group *A* = *i*, *B* = *j*, *C* = *k*. To indicate the mean of means across a dimension we write a star (⋆) in the corresponding index. Thus, in the example µij⋆ is the mean of the groups *A* = *i*, *B* = *j*, *C* = 1, 2, 3, 4. To simplify the discussion we assume that the grand mean µ⋆⋆⋆ over all groups is zero. This can always be achieved by subtracting a given non-zero grand mean from each group mean. In testing the main effects, the null hypothesis is that all means of the corresponding factor are identical.   
  
For the main effect of factor *A* the hypotheses are, for instance:

H0 : µ1⋆⋆ = µ2⋆⋆ = µ3⋆⋆   
H1 : µi⋆⋆ ≠ µj⋆⋆ for at least one index pair *i*, *j*.

The assumption that the grand mean is zero implies that ∑i µi⋆⋆ = ∑j µ⋆j⋆ = ∑k µ⋆⋆k = 0. The above hypotheses are therefore equivalent to

H0 : µi⋆⋆ = 0 for all *i,*   
H1 : µi⋆⋆ ≠ 0 for at least one *i*.

In testing two-factor interactions, the residuals δij⋆, δi⋆k, and δ⋆ik of the groups means after subtraction of the main effects are considered. For the A × B interaction of the example, the 3 × 3 = 9 relevant residuals are δij⋆ = µij⋆ − µi⋆⋆ − µ⋆j⋆. The null hypothesis of no interaction effect states that all residuals are identical. The hypotheses for the A × B interaction are, for example:

H0 : δij⋆ = δkl⋆ for all index pairs *i*, *j* and *k*, *l*.   
H1 : δij⋆ ≠ δkl⋆ for at least one combination of *i*, *j* and *k*, *l*.

The assumption that the grand mean is zero implies that ∑i,j δij⋆ = ∑i,k δi⋆k = ∑j,k δ⋆jk = 0. The above hypotheses are therefore equivalent to

H0 : δij⋆ = 0 for all *i*, *j*   
H1 : δij⋆ ≠ 0 for at least one *i*, *j*.

In testing the three-factor interactions, the residuals δijk of the group means after subtraction of all main effects and all two-factor interactions are considered. In a three factor design there is only one possible three-factor interaction. The 3 × 3 × 4 = 36 residuals in the example are calculated as δijk = µijk − µi⋆⋆ − µ⋆j⋆ − µ⋆⋆k − δij⋆ − δi⋆k − δ⋆jk. The null hypothesis of no interaction states that all residuals are equal. Thus,

H0 : δijk = δlmn for all combinations of index triples *i*, *j*, *k* and *l*, *m*, *n*.   
H1 : δijk ≠ δlmn for at least one combination of index triples *i*, *j*, *k* and *l*, *m*, *n*.

The assumption that the grand mean is zero implies that ∑i,j,k δijk = 0. The above hypotheses are therefore equivalent to

H0 : δijk = 0 for all *i*, *j*, *k*   
H1 : δijk ≠ 0 for at least one *i*, *j*, *k.*

It should be obvious how the reasoning outlined above can be generalized to designs with 4 and more factors.

**Planned comparisons**

Planned comparison are speciﬁc tests between levels of a factor planned before the experiment was conducted. One application is the comparison between two sets of levels of a factor. The general idea is to subtract the means across two sets of levels that should be compared from each other and to test whether the difference is zero. Formally this is done by calculating the sum of the componentwise product of the mean vector ⃗µ and a nonzero contrast vector ⃗*c* (i.e. the scalar product of ⃗µ and ⃗c): C = ∑i=1..k *c*i µi. The contrast vector ⃗*c* contains negative weights for levels on one side of the comparison, positive weights for the levels on theother side of the comparison, and zero for levels that are not part of the comparison. The sum of weights is always zero. Assume, for instance, that we have a factor with 4 levels and mean vector ⃗µ = (2, 3, 1, 2). Further assume that we want to test whether the means in the ﬁrst two levels are identical to the means in the last two levels. In this case we deﬁne ⃗c = (−1/2, −1/2, 1/2, 1/2) and get C = ∑i ⃗µi⃗ci = −1 − 3/2 + 1/2 + 1 = −1.   
  
A second application is the testing of polygonal contrasts in a trend analysis. In this case it is normally assumed that the factor represents a quantitative variable and that the levels of the factor that correspond to speciﬁc values of this quantitative variable are equally spaced (for more details, see e.g. Hays (1988, p. 706ff)). In a factor with *k* levels, *k* − 1 orthogonal polynomial trends can be tested.   
  
In planned comparisons the null hypothesis is: H0 : *C* = 0, and the alternative hypothesis H1 : *C* ≠ 0.

**Effect size index**

The effect size *f* is deﬁned as: *f* = σm/σ. In this equation σm is the standard deviation of the group means µi and σ the common standard deviation within each of the *k* groups. The total variance is then σ2t = σ2m + σ2. A different but equivalent way to specify the effect size is in terms of η2, which is deﬁned as η2 = σ2m/σ2t. That is, η2 is the ratio of the between-groups variance σ2m and the total variance σ2t and can be interpreted as the proportion of variance explained by the group membership. The relationship between η2 and *f* is:

η2 = *f*2/(1 + *f*2)

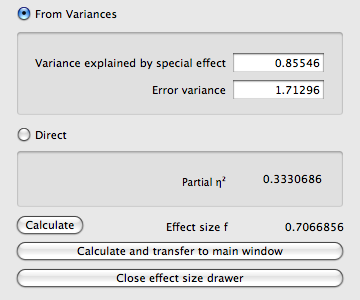
or, if solved for *f*,

*f* = √(η2 /(1 − η2)).

Cohen (1969, p.348) deﬁned the following effect size conventions:

small *f* = 0.10   
medium *f* = 0.25   
large *f* = 0.40

Pressing the *Determine* button to the left of the effect size label opens the effect size drawer. You can use this drawer to calculate the effect size *f* from variances or from η2.   
  
If you choose *From Variances* then you need to insert the variance explained by the effect under consideration, that is, σ2m into the *Variance explained by special effect* ﬁeld, and the square of the common standard deviation within each group, that is, σ2, into the *Variance within groups ﬁeld*. Alternatively, you may choose the *Direct* option and then specify the effect size *f* via η2.

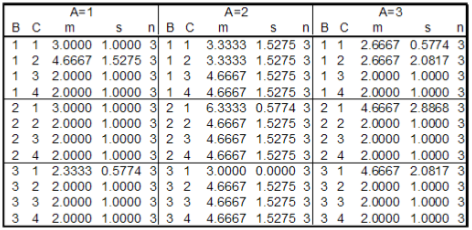


**Options**

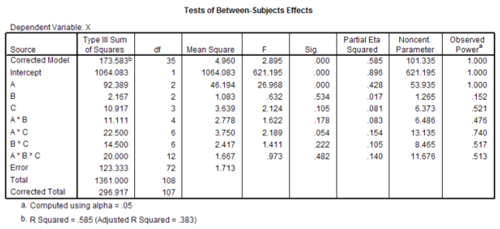
This test has no options.

**Examples**

To illustrate the test of main effects and interaction we assume the speciﬁc values for an A × B × C design as illustrated here:



Note, however, that we use this sample data example for didactical purposes only. In G\*Power, the input consists of population parameters, not sample statistics.  
  
The next table shows the results of a SPSS analysis (GLM univariate) performed on these data. We will show how to reproduce the values in the *Observed Power* column in the SPSS output with G\*Power.



As a ﬁrst step we calculate the grand mean of the data. Since all groups have the same size (*n* = 3) this is just the arithmetic mean of all 36 groups means: *m*g = 3.1382. We then subtract this grand mean from all cells (this step is not essential but makes the calculation and discussion easier). Next, we estimate the common variance σ within each group by calculating the mean variance of all cells, that is, σ2 = 1/36 ∑i s2i = 1.71296.

**Main effects**

In order to calculate the power for the *A*, *B*, and *C* main effects we need to know the effect size *f* = σm/σ. We already know σ2 to be 1.71296. We still need to calculate the variance of the means σ2m for each factor. The procedure is analogous for all three main effects. We therefore demonstrate only the calculations necessary for the main effect of factor *A*.  
  
We ﬁrst calculate the three means for factor *A*: µi⋆⋆ = {−0.722231, 1.30556, −0.583331}. Due to the fact that we have ﬁrst subtracted the grand mean from each cell we have ∑i µi⋆⋆ = 0, and we can easily compute the variance of these means as mean square: σ2m = 1/3 ∑i µ2i⋆⋆ = 0.85546. With these values we calculate *f* = √(σm/σ) = √(0.85546/1.71296) = 0.7066856.   
  
The effect size drawer in G\*Power can be used to do the last calculation: We choose *From Variances* and insert 0.85546 in the *Variance explained by special effect* and 1.71296 in the *Error variance* ﬁeld. Pressing the *Calculate* button gives the above value for *f* (i.e., 0.7066856) and a partial η2 of 0.3330686. Note that the partial η2 given by G\*Power is calculated from *f* according to the following formula:

η2 = *f*2 /(1 + *f*2)

It is not identical to the SPSS partial η2, which is based on sample estimates. The relation between the two is

"SPSS η20" = η2 *N*/(*N* + *k*(η2 − 1)),

where *N* denotes the total sample size, *k* the total number of groups in the design and η2 the G\*Power value. Thus, η20 = 0.33306806 · 108/(108 + 0.33306806 · 36 − 36) = 0.42828, which is the value given in the SPSS output.  
  
We now use G\*Power to calculate the power for α = 0.05 and a total sample size of 3 · 3 · 4 · 3 = 108. We set

**Select**

Type of power analysis: Post hoc

**Input**

Effect size f : 0.7066856   
α err prob: 0.05   
Total sample size: 108   
Numerator df: 2 (number of factor levels - 1, A has 3 levels)   
Number of groups: 36 (total number of groups in the design)

**Output**

Noncentrality parameter λ: 53.935690   
Critical F: 3.123907   
Denominator df: 72   
Power (1-β err prob): 0.99999

The value of the noncentrality parameter and the power computed by G\*Power are identical to the values in the SPSS output.

**Two-factor interactions**

To calculate the power for two-factor interactions *A* × *B*, *A* × *C*, and *A* × *B* we need to calculate the effect size *f* corresponding to the values given in the table displayed above. The procedure is analogous for each of the three two-factor interactions. We thus restrict ourselves to the *A* × *B* interaction.  
  
The values needed to calculate σ2m are the 3 × 3 = 9 residuals δij⋆ . They are given by δij⋆ = µij⋆ − µi⋆⋆ −µ⋆j⋆ = {0.555564, -0.361111, -0.194453, -0.388903, 0.444447, -0.0555444, -0.166661, -0.0833361, 0.249997}. The mean of these values is zero (as a consequence of subtracting the grand mean). Thus, the variance σm is given by 1/9 ∑i,j δ2ij⋆ = 0.102881. This results in an effect size of *f* = √(0.102881/1.71296) = 0.2450722 or, in terms of the proportion of variance explained, in η2 = 0.0557195. Using the formula given in the previous section on main effects it can be checked that this corresponds to an "SPSS η20" of 0.0813, which is identical to the value given   
in the SPSS output.  
  
We use G\*Power to calculate the power for α = 0.05 and a total sample size of 3 × 3 × 4 × 3 = 108. We set:

**Select**

Type of power analysis: Post hoc

**Input**

Effect size f : 0.2450722   
α err prob: 0.05   
Total sample size: 108   
Numerator df: 4 (#A-1)(#B-1) = (3-1)(3-1)   
Number of groups: 36 (total number of groups in the design)

**Output**

Noncentrality parameter λ: 6.486521   
Critical F: 2.498919   
Denominator df: 72   
Power (1-β err prob): 0.475635

(The notation #A in the comment above means number of levels in factor A). A check reveals that the value of the noncentrality parameter and the power computed by G\*Power are identical to the values for (A \* B) in the SPSS output.

**Three-factor interations**

To calculate the effect size of the three-factor interaction corresponding to the values given in table 7 we need the variance of the 36 residuals δijk = µijk − µi⋆⋆ − µ⋆j⋆ − µ⋆⋆k − δij⋆ − δi⋆j − δ⋆jk = {0.333336, 0.777792, -0.555564, -0.555564, -0.416656, -0.305567, 0.361111, 0.361111, 0.0833194, - 0.472225, 0.194453, 0.194453, 0.166669, -0.944475, 0.388903, 0.388903, 0.666653, 0.222242, -0.444447, -0.444447, -0.833322, 0.722233, 0.0555444, 0.0555444, -0.500006, 0.166683, 0.166661, 0.166661, -0.249997, 0.083325, 0.0833361, 0.0833361, 0.750003, -0.250008, - 0.249997, -0.249997}. The mean of these values is zero (as a consequence of subtracting the grand mean). Thus, the variance σm is given by 1/36 ∑i,j,k δ2ijk = 0.185189. This results in an effect size *f* = √(0.185189/1.71296) = 0.3288016 which is equivalent to η2 = 0.09756294.   
  
Using the formula given in the previous section on main effects it can be checked that this corresponds to an "SPSS η2" of 0.140, which is identical to that given in the SPSS output.

We use G\*Power to calculate the power for α = 0.05 and a total sample size of 3 × 3 × 4 × 3 = 108. We set:

**Select**

Type of power analysis: Post hoc

**Input**

Effect size f : 0.3288016   
α err prob: 0.05   
Total sample size: 108   
Numerator df: 12 (#A-1)(#B-1)(#C-1) = (3-1)(3-1)(4-1)   
Number of groups: 36 (total number of groups in the design)

**Output**

Noncentrality parameter λ: 11.675933   
Critical F: 1.889242   
Denominator df: 72   
Power (1-β err prob): 0.513442

(The notation #A in the comment above means number of levels in factor A). Again a check reveals that the value of the noncentrality parameter and the power computed by G\*Power are identical to the values for (*A* \* *B* \* *C*) in the SPSS output.

U**sing conventional effect sizes**

In the example given in the previous section, we assumed that we know the true values of the mean and variances in all groups. We are, however, rarely in such a privileged position. Instead, we usually only have rough estimates of the expected effect sizes. In these cases we may resort to the conventional effect sizes proposed by Cohen.   
  
Assume that we want to calculate the total sample size needed to achieve a power of 0.95 in testing the *A* × *C* two-factor interaction at α level 0.05. Assume further that the total design in this scenario is *A* × *B* × *C* with 3 × 2 × 5 factor levels, that is, 30 groups. Theoretical considerations suggest that there should be a small interaction. We thus use the conventional value *f* = 0.1 deﬁned by Cohen (1969) as small effect. The inputs into and outputs of G\*Power for this scenario are:

**Select**

Type of power analysis: A priori

**Input**

Effect size f : 0.1  
α err prob: 0.05   
Power (1-β err prob): 0.95  
Numerator df: 8 (#A-1)(#C-1) = (3-1)(5-1)  
Number of groups: 30

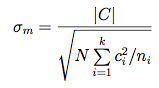
**Output**

Noncentrality parameter λ: 22.830000   
Critical F: 1.942507   
Denominator df: 2253   
Total sample size: 2283  
Power (1-β err prob): 0.950078

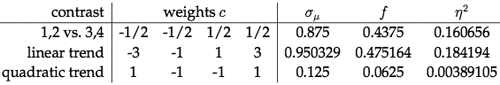
(The notation #A in the comment above means number of levels in factor A) G\*Power calculates a total sample size of 2283. Please note that this sample size is not a multiple of the group size 30 (2283/30 = 76.1)! If you want to ensure that your have equal group sizes, round this value up to a multiple of 30 by chosing a total sample size of 30\*77= 2310. A post hoc analysis with this sample size reveals that this increases the power to 0.952674.

**Power for planned comparisons**

To calculate the effect size *f* = σm/σ for a given comparison C = ∑i=1.. k µi *c*i we need to know both the standard deviation σ within each group and the standard deviation σm of the effect. The latter is given by



where *N, n*i denote total sample size and sample size in group *i*, respectively. Given the mean vector µ = (1.5, 2, 3, 4), equal sample sizes of *n*i = 5 in each group, and standard deviation σ = 2 within each group, we want to calculate the power for the following contrasts:



Each contrast has a numerator df = 1. The denominator dfs are *N* - *k*, where *k* is the number of levels (4 in the example). To calculate the power of the linear trend at α = 0.05 we specify:

**Select**

Type of power analysis: Post hoc

**Input**

Effect size f : 0.475164   
α err prob: 0.05   
Total sample size: 20  
Numerator df: 1  
Number of groups: 4

**Output**

Noncentrality parameter λ: 4.515617  
Critical F: 4.493998   
Denominator df: 16   
Power (1-β err prob): 0.514736

Inserting the *f* 's for the other two contrasts yields a power of 0.451898 for the comparison of "1,2 vs. 3,4", and a power of 0.057970 for the test of a quadratic trend.

Related tests

[ANOVA: Fixed effects, omnibus, one-way](http://www.psycho.uni-duesseldorf.de/abteilungen/aap/gpower3/user-guide-by-distribution/f/anova_fixed_effects_omnibus)

**Implementation notes**

The distribution under H0 is the central *F* (*df*1, *N* − *k*) distribution. The numerator *df*1 is speciﬁed in the input and the denominator df is *df*2 = *N* − *k*, where *N* is the total sample size and *k* the total number of groups in the design. The distribution under H1 is the noncentral *F* (*df*1, *N* − *k,* λ) distribution with the same df 's and noncentrality parameter λ = *f*2 *N*.

**Validation**

The results were checked against the values produced by GPower 2.0.